Sample Question Paper - 1
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Solve: $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$ **[2]**

OR

Determine whether the given values are solutions of the given equation or not:

$x^2 - 3x + 2 = 0$, x = 2, x = -1

2. Eight solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 6 cm and height 32 cm. Then, find the diameter of each sphere. **[2]**

3. If the mean of the following data is 20.6. Find the value of p. **[2]**

x	10	15	p	25	35
f	3	10	25	7	5

4. The sum of n terms of an AP is $3n^2 + 5n$. Find the AP. Hence, find its 16^{th} term. **[2]**

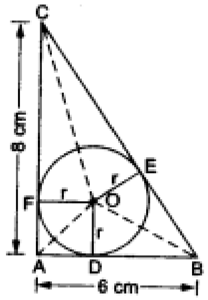
5. The frequency distribution of agricultural holdings in a village is given below : **[2]**

Area of land [in hectare)	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11	11 -13
Number of families	20	45	80	55	40	12

Find the modal agricultural holdings of the village

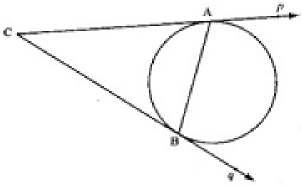
6. In the given figure, ABC is a right-angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of the radius of the **[2]**

inscribed circle.



OR

Prove that the tangents drawn at the end of a chord of a circle make equal angle with the chord.



Section B

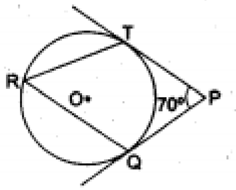
7. In an AP: $a_n = 4$, $d = 2$, $S_n = -14$, find n and a. **[3]**

8. The angle of elevation of a cloud from a point 200 m above the lake is 30^0 and the angle of depression of its reflection in the lake is 60^0, find the height of the cloud above the lake. **[3]**

OR

A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall.

9. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, find $\angle TRQ$. **[3]**



10. Find roots of given quadratic equation: $p^2x^2 + \left(p^2 - q^2\right)x - q^2 = 0, p \neq 0$ **[3]**

Section C

11. Determine a point which divides a line segment of length 12 cm internally in the ratio 2: 3. Also, justify your construction. **[4]**

OR

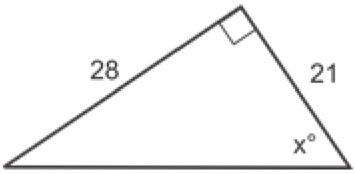
Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

12. The following table shows the ages of the patients admitted in a hospital during a year: **[4]**

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

13. Tushar a class 10th student after reading the chapter Application of trigonometry started solving the problems of the chapter. He was solving different types of questions, one of them is related to triangle as shown in the figure below. **[4]**



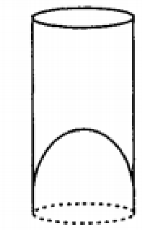
By analysing the above given figure answer the following questions:

i. He writes $x^0 = \tan^{-1}(\frac{21}{28})$

He made a mistake. What did he do wrong?

ii. How could Tushar have found the hypotenuse in the triangle avoiding making mistake?

14. A juice seller was serving his customers using glasses as shown in Fig. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. **[4]**



i. If the height of a glass was 10 cm, find the apparent capacity of the glass

ii. Also, find its actual capacity. (Use π = 3.14.)

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. We have ,$\sqrt{7}x^2-6x-13\sqrt{7}=0 \Rightarrow \sqrt{7}x^2-13x+7x-13\sqrt{7}=0$
$\Rightarrow x(\sqrt{7}x-13)+\sqrt{7}(\sqrt{7}x-13)=0$
$\Rightarrow$ (x + $\sqrt{7}$)($\sqrt{7}x$- 13) = 0
$\Rightarrow$ x + $\sqrt{7}$ = 0 or $\sqrt{7}x$ - 13 = 0
x = -$\sqrt{7}$ or $x=\frac{13}{\sqrt{7}}=\frac{13\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}}=\frac{13\sqrt{7}}{7}$.

OR

We have the following equation,
$x^2-3x+2=0$
Putting x = 2, we get LHS = 2^2 - 3(2) + 2
= 4 - 6 + 2
= 0 = RHS
x^2 - 3x + 2 = 0,x = 2 is a solution of the equation.
Now, putting x = -1, we get
LHS= $(-1)^2$ - 3(-1) + 2
= 1 + 3 + 2
= 6 $\neq$ RHS
x^2 - 3x + 2 = 0, x = -1 is not a solution of x^2 - 3x + 2 = 0

2. Metallic cylinder is melted to form 8 solid spheres.
Diameter of cylinder = 6 cm
So Radius of cylinder R = 3 cm
Height of cylinder = 32 cm
So, diameter (d) of sphere = ?
Volume of cylinder = 8 $\times$ volume of spheres
$\Rightarrow \pi R^2h=8\times\frac{4}{3}\pi r^3$
$\Rightarrow 3^2\times 32=\frac{8\times 4}{3}\times r^3$
$\Rightarrow \frac{9\times 32\times 3}{8\times 4}$ = r^3
$\Rightarrow$ r^3 = 27
$\therefore$ r = 3 cm.
Hence diameter of each sphere, d = 3 $\times$ 2 = 6 cm

3.

X	**f**	**fx**
10	3	30
5	10	150
p	25	25p
25	7	175
35	5	175
	N = 50	**Sum = 530 + 25p**

According to the question , Mean = 20.6
$\sum f_x$ = 530 + 25 p
and $\sum$ f = 50
We know that, $\bar{x}=\frac{\sum fx}{\sum f}$
$\Rightarrow$ 20.6 = $\frac{(530+25p)}{50}$

$\Rightarrow$ 25 p = 500
$\Rightarrow$ p = 20

4. Given: $S_n = 3n^2 + 5n$

$\therefore S_1 = 3(1)^2 + 5(1) = 8$

$\Rightarrow a_1 = 8$ ..(i)

$S_2 = 3(2)^2 + 5(2) = 22$

$\Rightarrow a_1 + a_2 = 22$

$\Rightarrow 8 + a_2 = 22$ [using (i)]

$\Rightarrow a_2 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

AP is 8, 14, 20, 26, ...
Now, $a_{16} = a + 15d$

$= 8 + 15(6) = 98$

5. Modal class = 5 - 7
Here, $l = 5, f_1 = 80, f_0 = 45, h = 2, f_2 = 55$

Mode = $l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$

$= 5 + \frac{(80-45)}{2(80)-45-55} \times 2$

$= 5 + \frac{80-45}{160-45-55} \times 2$

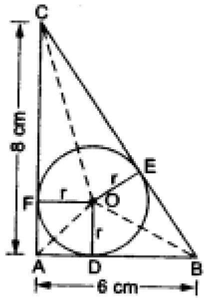
$= 5 + \frac{35}{160-100} \times 2$

$= 5 + \frac{35 \times 2}{60}$

$= 5 + \frac{70}{60}$

$= 5 + 1.17$

$= 6.17$

6. 

Join OA, OB and OC.
Draw $OD \perp AB, OE \perp BC$
and $OF \perp CA$
Then, OD = OE = OF = r cm.

$\therefore \quad \text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times AC$

$= \left(\frac{1}{2} \times 6 \times 8\right) \text{cm}^2 = 24\text{cm}^2$

Now, $\text{ar}(\Delta ABC) = \frac{1}{2} \times (\text{ perimeter of } \Delta ABC) \times r$

$\Rightarrow \quad 24 = \frac{1}{2} \times (AB + BC + CA) \times r$

$\Rightarrow \quad 24 = \frac{1}{2} \times (6 + 10 + 8) \times r$

$\Rightarrow r = 2 \left[\because BC^2 = AB^2 + AC^2 \Rightarrow BC = \sqrt{6^2 + 8^2} = 10\right]$

Hence, the radius of the inscribed circle is 2cm.

OR

Let AB be a chord of the circle and P and Q are the tangents to the circle at A and B respectively.
Let the tangent P and Q, when produced meet at C.
Now, CA and CB are tangent to the circle at A and B from an external point C.
$\therefore$ CA = CB [lengths of tangents from P are equal]

In $\triangle$ACB,

$\angle$ CAB = $\angle$CBA [$\because$ In a triangle angles opposite to equal sides are equal]

Section B

7. Here, $a_n = 4$

d = 2

$S_n = -14$

We know that

$a_n = a + (n - 1)d$

$\Rightarrow$ 4 = a + (n - 1)d

$\Rightarrow$ 4 = a + 2n - 2

$\Rightarrow$ 4 + 2 = a + 2n

$\Rightarrow$ 6 = a + 2n

$\Rightarrow$ a + 2n = 6 ...... (1)

Again, we know that

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow -14 = \frac{n}{2}[2a + (n - 1)2]$

$\Rightarrow$ -14 = n[a + (n - 1)]

$\Rightarrow$ -14 = n (a + n - 1)

$\Rightarrow$ -14 = n (6 - n - 1) ....From (1), (a + 2n = 6 $\Rightarrow$ a + n = 6 - n)

$\Rightarrow$ -14 = n(-n + 5)

$\Rightarrow$ -14 = $-n^2$ + 5n

$\Rightarrow$ n^2 - 7n + 2n - 14 = 0

$\Rightarrow$ n(n - 7) + 2(n - 7) = 0

$\Rightarrow$ (n - 7) (n + 2) = 0

$\Rightarrow$ n - 7 = 0 or n + 2 = 0

$\Rightarrow$ n = 7 or n = -2

$\Rightarrow$ n = - 2 is in admissible as n, being the number of terms, is a natural number.

$\therefore$ n = 7

Putting n = 7 in equation (1), we get

a + 2(7) = 6

$\Rightarrow$ a + 14 = 6

$\Rightarrow$ a = 6 - 14

$\Rightarrow$ a = -8

8. In $\triangle$ADC,

$\tan 30^\circ = \frac{H-200}{x}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{H-200}{x}$

$\Rightarrow \quad x = \sqrt{3}$(H - 200) m. ....(1)

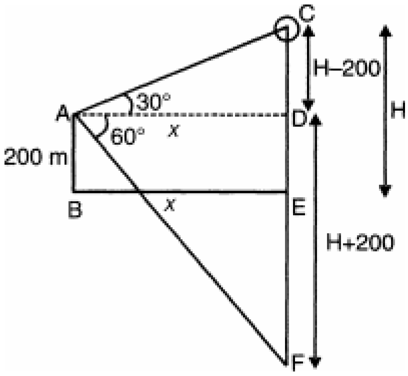
In $\triangle$ADF,

$\tan 60^\circ = \frac{H+200}{x}$

$\sqrt{3} = \frac{H+200}{x}$

$\sqrt{3}x = H + 200$...............(2)

here H is height of cloud above the lake.



Substituting the value of x from (1) into (2) we get

3(H - 200) = H + 200

3H - 600 = H + 200

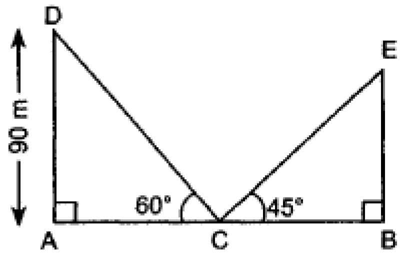
2H = 800

H = 400 m

So height of the cloud above the lake is 400 m.

OR

Let AB is path



In rt. △DAC, $\frac{DC}{AD} = cosec\ 60^{\circ}$

$\Rightarrow \quad \frac{\mathrm{DC}}{90} = \frac{2}{\sqrt{3}}$

$\mathrm{DC} = \frac{2}{\sqrt{3}} \times 90\mathrm{m} = \frac{180}{\sqrt{3}}\mathrm{m}$

Now, $DC = CE$

$\therefore \quad \mathrm{CE} = \frac{180}{\sqrt{3}}\mathrm{m}$

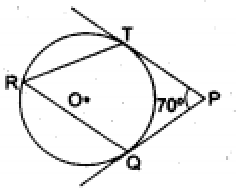
In rt. △EBC,

$\frac{BE}{CE} = \sin 45^{\circ}$

$\Rightarrow \quad \mathrm{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}\mathrm{m}$

$\Rightarrow \quad BE = 73.47\mathrm{m}$

9. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, then,we have to find $\angle TRQ$.



We know that the radius and tangent are perpendicular at their point of contact.

$\angle OTP = \angle OQP = 90^{\circ}$

Now, In quadrilateral OQPT

$\angle QOT + \angle OTP + \angle OQP + \angle TPQ = 360^{\circ}$ [Angle sum property of a quadrilateral]

$\angle QOT + 90^{\circ} + 90^{\circ} + 70^{\circ} = 360^{\circ}$

$250^{\circ} + \angle QOT = 360^{\circ}$

$\angle QOT = 110^{\circ}$

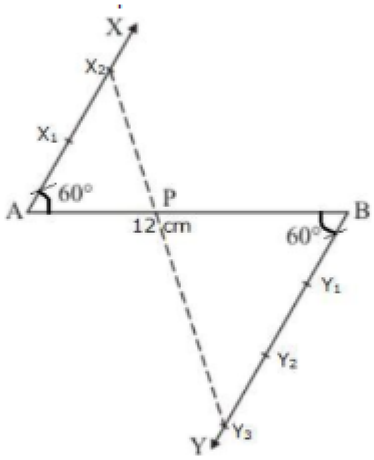
We know that the angle subtended by an arc at the centre is double of the angle subtended by the arc at any point on the circumference of the circle.

$\angle TRQ = \frac{1}{2}\angle QOT \Rightarrow \angle TRQ = \frac{1}{2} \times 110^{\circ} = 55^{\circ}$

10. We have, $p^2x^2+(p^2-q^2)x-q^2=0$
Comparing this equation with $ax^2+bx+c=0$, we have
$a=p^2, b=p^2-q^2 \; and \; c=-q^2$
$\therefore$ D=b^2 - 4ac = $(p^2- q^2)^2$- $4p^2(-q^2)$
= $(p^2- q^2)^2 + 4p^2q^2$
=$(p^2+q^2)^2>0$
So, the given equation has real roots.
Let the roots of given equation are $\alpha \; and \; \beta$
$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(p^2-q^2)+(p^2+q^2)}{2p^2}=\frac{q^2}{p^2}$
and, $\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(p^2-q^2)-(p^2+q^2)}{2p^2}=-1$
$\therefore x=-1$ or $x=\frac{q^2}{p^2}$

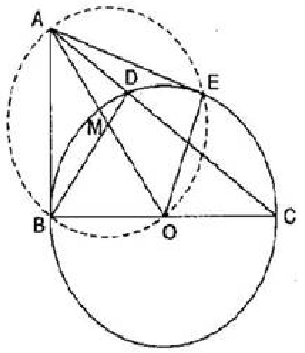
Section C

11. Steps of construction



i. Draw a line segment AB of 12 cm
ii. Through the points A and B draw two parallel line on the opposite side of AB
iii. Cut 2 equal parts on AX and 3 equal parts on BY such that
$AX_1 = X_1X_2$ and BY1 = $Y_1Y_2 = Y_2Y_3$
iv. Join X_2Y_3 which intersect AB at P
$\therefore \frac{AP}{PB}=\frac{2}{3}$
Justification:-
In $\triangle AX_2P$ and $\triangle BY_3P$ [vertically opposite angles]
$\angle APX_2=\angle BPY_3$ [alternate interior angles]
$\angle X_2AP=\angle Y_3BP$ [By AA similarity]
then $\triangle AX_2P \sim \triangle BY_3P$ (c.p.c.t)

OR



To construct : A right triangle ABC with AB = 6 cm, BC = 8 cm and $\angle B=90^\circ$.BD is the perpendicular from B on AC and the tangents from A to this circle.
Steps of Construction :
i. Draw a right triangle ABC with AB = 6 cm, BC = 8 cm and $\angle B=90^\circ$. Also, draw perpendicular BD on AC.

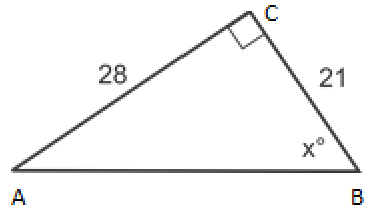
ii. Join AO and bisect it at M (here O is the centre of circle through B, C, D).
iii. Taking M as centre and MA as radius, draw a circle. Let it intersects the given circle at the points B and E.
iv. Join AB and AE.
Then AB and AE are the required two tangents.
Justification: Join OE.
Then, $\angle AEO$ is an angle in the semicircle.
$\Rightarrow \angle AEO = 90^\circ \Rightarrow AE \perp OE$
Since OE is a radius of the given circle, AE has to be a tangent to the circle. Similarly, AB is also a tangent to the circle.

12. Mode:
Here, the maximum frequency is 23 and the class corresponding to this frequency is 35 - 45.
So, the modal class is 35 - 45.
Now, size (h) = 10
lower limit it (l) of modal class = 35
frequency (f_1) of the modal class = 23
frequency (f_0) of class previous the modal class = 21
frequency (f_2) of class succeeding the modal class = 14
$\therefore$ Mode = l + $\frac{f_1-f_0}{2f_1-f_0-f_2} \times$ h = 35 + $\frac{23-21}{2\times23-21-14} \times 10$
= 35 + $\frac{2}{11} \times$ 10 = 35 + $\frac{20}{11}$
= 35 + 1.8 (approx.)
= 36.8 years (approx.)
Mean:-
Take a = 40, h = 10.

Age (in years)	**Number of patients (f_i)**	**Class marks (x_i)**	**$d_i = x_i - 40$**	$u_i = \frac{x_i-40}{10}$	**f_iu_i**
5-15	6	10	–30	–3	–18
15-25	11	20	–20	–2	–22
25-35	21	30	–10	–1	–21
35-45	23	40	0	0	0
45-55	14	50	10	1	14
55-65	5	60	20	2	10
Total	$\sum f_i$ = 80				$\sum f_iu_i$ = - 37

Using the step deviation method,
$\overline{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times$ h = 40 + $\left(\frac{-37}{80}\right) \times 10$
= 40 - $\frac{37}{8}$ = 40 - 4.63
= 35.37 years
Interpretation:- Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

13. i. In trigonometry, a tangent of an angle is the ratio of the perpendicular to the base of the triangle.
Here, he has written $x^o = \tan^{-1}(\frac{21}{28})$ which is the ratio of base to the perpendicular. Hence, it is wrong.
The correct solving is $x^o = \tan^{-1}(\frac{28}{21})$.
ii. Now, correct $x^o = \tan^{-1}(\frac{28}{21}) = 53.13^o$
The above-given figure can be redrawn as shown below:



Here AB is the hypotenuse of the triangle.
Using trigonometry formula,
sin x = $\frac{AC}{AB}$ = $\frac{28}{AB}$
AB = $\frac{28}{\sin 53.13}$ = 35
Thus, the hypotenuse of the triangle is 35 units.

14. We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm
    i. The apparent capacity of the glass = Volume of cylinder
    = $\pi r^2 h$
    = $3.14 \times \left(\frac{5}{2}\right)^2 \times 10$
    $= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$
    ii. The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass
    The volume of hemispherical part = $\frac{2}{3}\pi r^2 h$ = $\frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$
    Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3